AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

A-level MATHEMATICS

Unit Pure Core 4

Friday 15 June 2018 Afternoon Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions.

Answer each question in the space provided for that question.

1 (a) Express $\dfrac{10 + 24x - 12x^2}{(3 - x)(1 + 4x)}$ in the form $A + \dfrac{B}{3 - x} + \dfrac{C}{1 + 4x}$, where A, B and C are integers.

[4 marks]

(b) Hence find $\displaystyle\int_0^2 \frac{10 + 24x - 12x^2}{(3 - x)(1 + 4x)}\,\mathrm{d}x$, giving your answer in the form $p + q\ln 3$.

[5 marks]

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2 The angle α is **acute** and $\cos\alpha = \frac{\sqrt{3}}{3}$. The angle β is **obtuse** and $\sin\beta = \frac{1}{3}$.

(a) Show that $\tan\alpha = \sqrt{2}$ and find an exact value for $\tan\beta$.

[3 marks]

(b) Hence show that $\tan(\alpha - \beta)$ can be written as $p\sqrt{2}$, where p is a rational number.

[2 marks]

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3 (a) Find the binomial expansion of $(1 - 9x)^{\frac{2}{3}}$ up to and including the term in x^2.

[2 marks]

(b) (i) Find the binomial expansion of $(64 - 9x)^{\frac{2}{3}}$ up to and including the term in x^2.

[3 marks]

(ii) Use your expansion from part **(b)(i)** to find an estimate for $67^{\frac{2}{3}}$, giving your answer in the form $p + \frac{q}{r}$ where p, q and r are positive integers with $q < r$.

[2 marks]

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4 The polynomial $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = 18x^3 - 3x^2 - 28x - 12$.

(a) (i) Use the Factor Theorem to show that $(3x + 2)$ is a factor of $\mathrm{f}(x)$.

[2 marks]

(ii) Express $\mathrm{f}(x)$ as a product of linear factors.

[3 marks]

(b) The function g is defined for all real values of θ by

$$\mathrm{g}(\theta) = 18\sin 2\theta \cos\theta - 3\cos 2\theta + 20\sin\theta + 27$$

(i) Show that the equation $\mathrm{g}(\theta) = 0$ can be written as $\mathrm{f}(x) = 0$, where $x = \sin\theta$.

[4 marks]

(ii) Hence solve the equation $\mathrm{g}(\theta) = 0$, giving your answers, in radians, to three significant figures in the interval $0 \leqslant \theta \leqslant 2\pi$.

[2 marks]

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5 In a conservation area, a disease is spreading amongst two species of wild animal, P and Q, which is reducing their numbers.

Previous experience has shown that the number of each of the species P and Q can be modelled by

$$\mathrm{p}(t) = 4500\mathrm{e}^{-\frac{1}{20}t} \quad \text{and} \quad \mathrm{q}(t) = 3000\mathrm{e}^{-\frac{1}{40}t} \text{ respectively}$$

where t is the time in weeks after the disease is first detected.

This outbreak of the disease was first detected on 1 May.

(a) Use the two models to find:

(i) the number of species P on 1 May;

[1 mark]

(ii) the number of species Q after 36 weeks from 1 May, giving your answer to the nearest 10;

[1 mark]

(iii) after how many weeks the number of species P will first fall below 1500.

[2 marks]

(b) Use logarithms and the two models to calculate the value of t when the number of species Q will be four times that of species P. Give your answer to the nearest whole number.

[3 marks]

(c) When $t = T$ the number of species Q first exceeds that of species P by 300.

(i) Use this information and the two models to derive a quadratic equation in x where $x = \mathrm{e}^{-\frac{1}{40}T}$.

[2 marks]

(ii) Hence find the number of days after 1 May when this difference of 300 animals will first occur. Give your answer to the nearest day.

[3 marks]

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6 A curve is defined by the equation

$$\cos 3y + y \sin^2 3x = x + k$$

The point $P\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ lies on this curve.

(a) Find the exact value of the constant k.

[1 mark]

(b) Find an expression for $\frac{dy}{dx}$.

[6 marks]

(c) Find the equation of the tangent to the curve at P, giving your answer in the form $y = mx + c$.

[2 marks]

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7 (a) Using a suitable substitution, or otherwise, find

$$\int \frac{x}{(7+2x^2)^2}\,\mathrm{d}x$$

[3 marks]

(b) Solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x\,\mathrm{e}^{4y}}{(7+2x^2)^2}$$

given that $y = 0$ when $x = 2$.

Give your answer in the form $y = \mathrm{f}(x)$.

[6 marks]

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8 The points A and C have coordinates $(3, -1, 2)$ and $(0, -1, -2)$ respectively.

The line l has equation $\mathbf{r} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix}$.

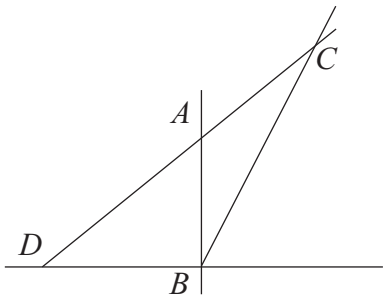
(a) (i) The point B lies on l where $\lambda = 2$. Find the coordinates of B.

[1 mark]

(ii) Find the **acute** angle ABC, giving your answer to the nearest $0.1°$.

[5 marks]

(b) The point D lies on a line through C and A such that angle ABD is a right angle.



The point E completes the rectangle $ABDE$. Find the coordinates of E.

[7 marks]

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END OF QUESTIONS

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